Non-Linear Inflation Dynamics

based on work with

Andres Blanco Corina Boar Callum Jones

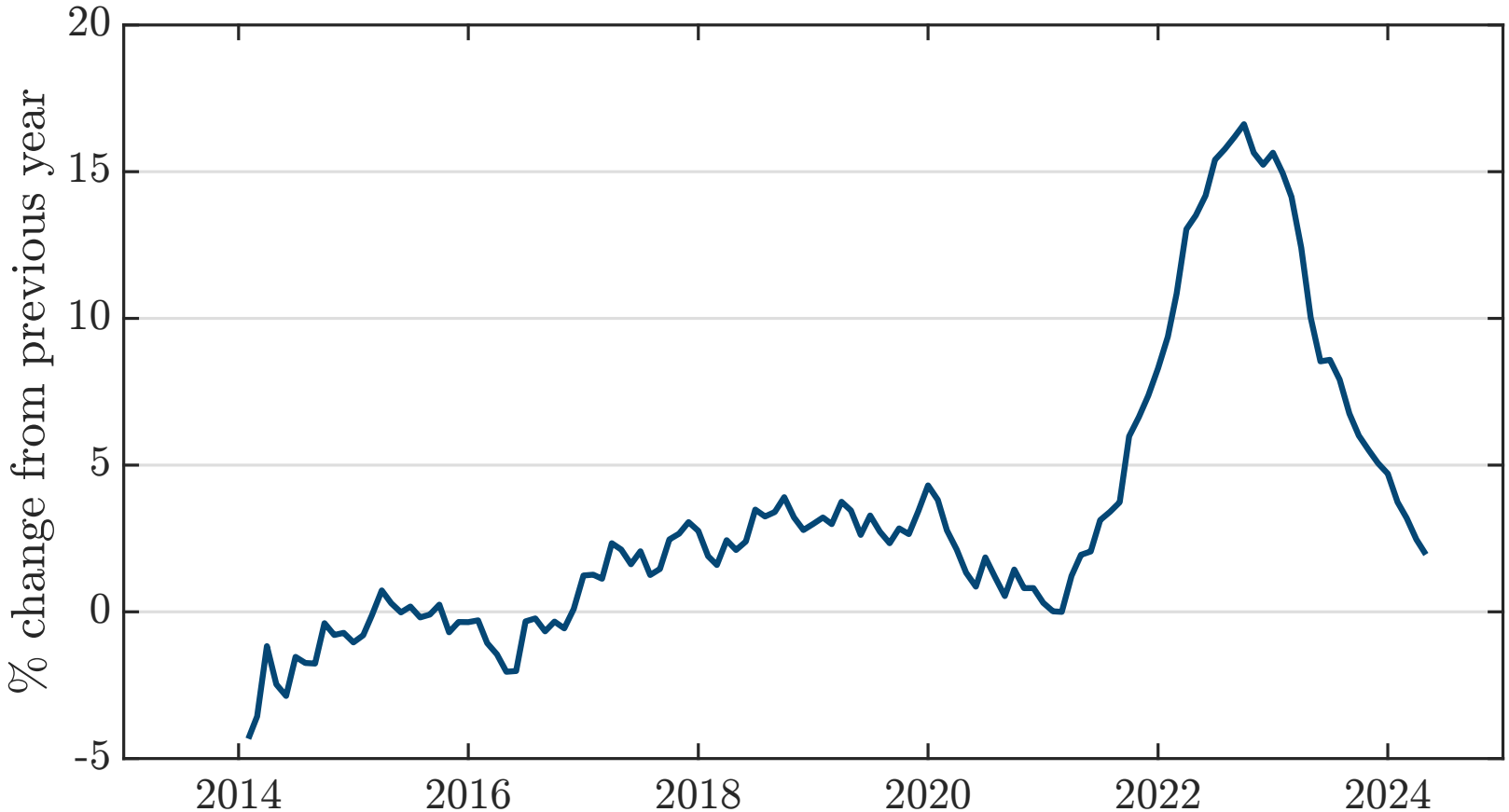
Bulgarian Council for Economic Analyses Annual Conference

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Motivation

- Large increase in inflation in many economies

Inflation in Bulgaria



Source: National Statistical Institute

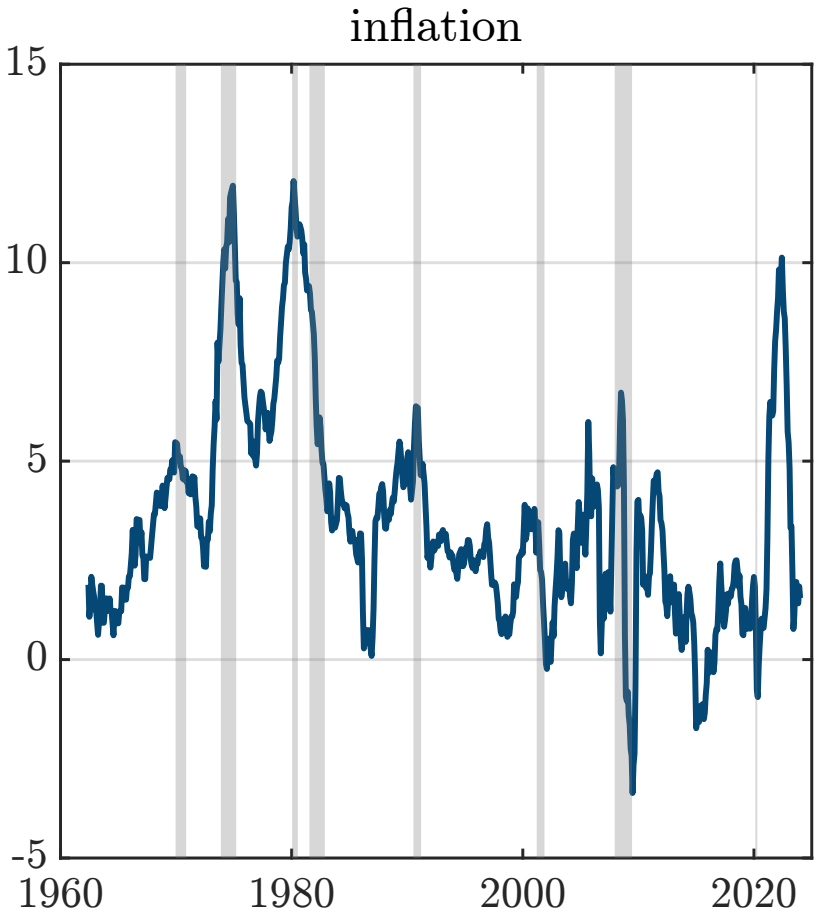
Motivation

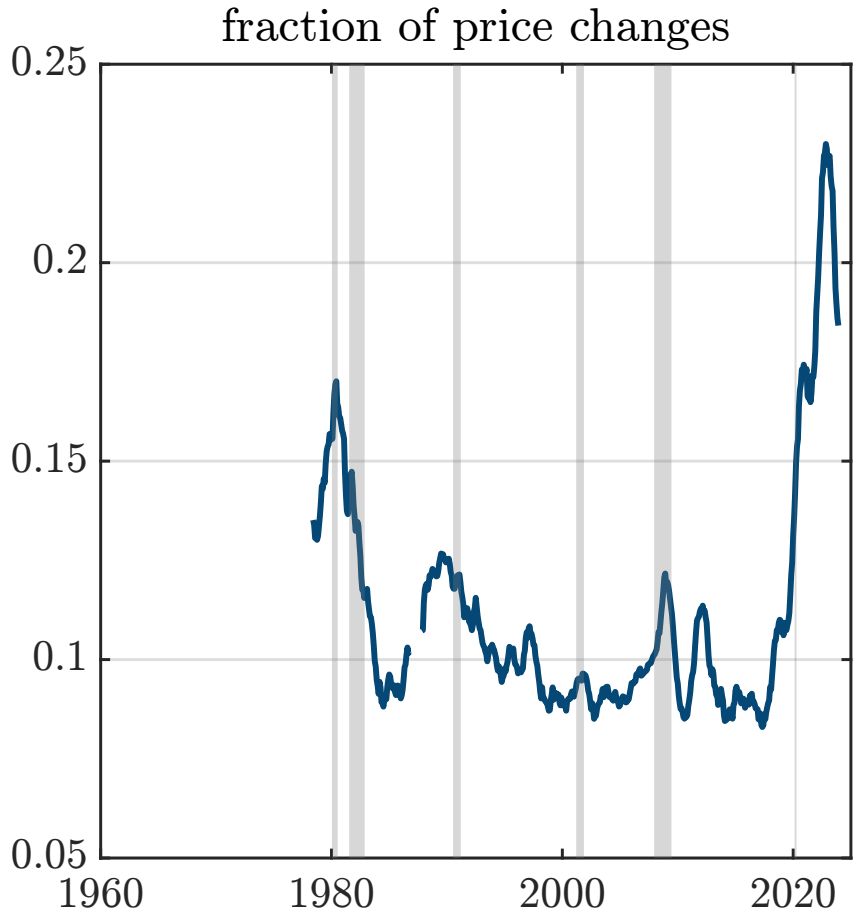
- Large increase in inflation in many economies
- Output cost of reducing inflation: slope of Phillips curve
- Baseline Calvo model: Phillips curve linear

$$\pi_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

- Slope of Phillips curve depends on how frequently prices change
  - in baseline Calvo model, slope $\sim$ fraction of price changes[2]
- Data: fraction of price changes increases with inflation

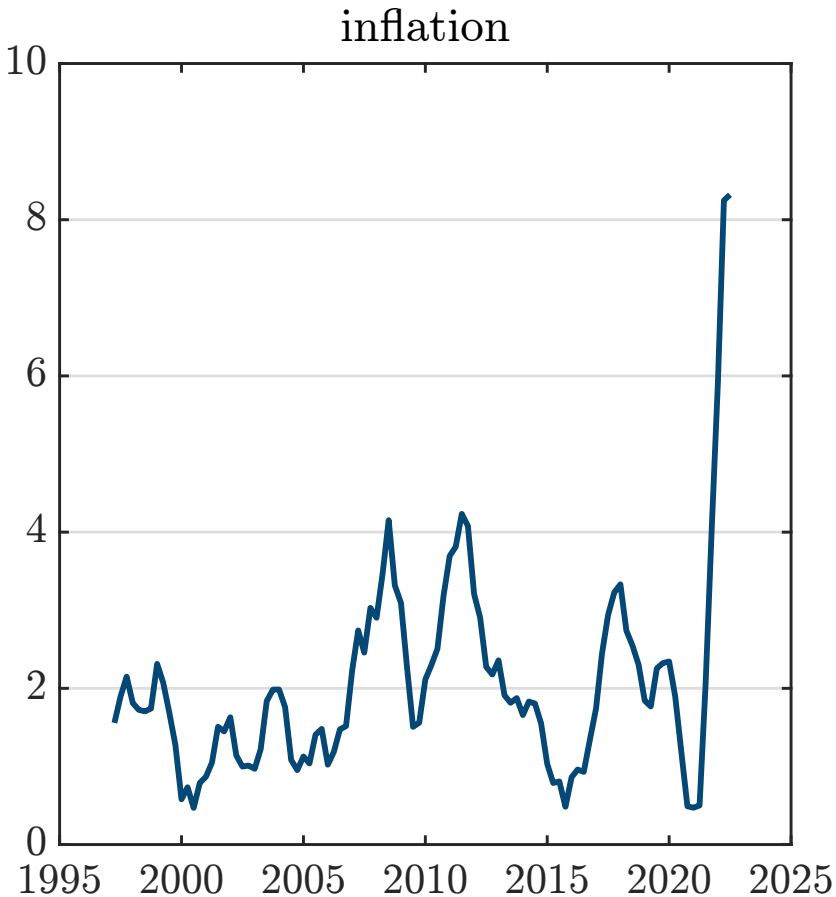
Evidence from the US

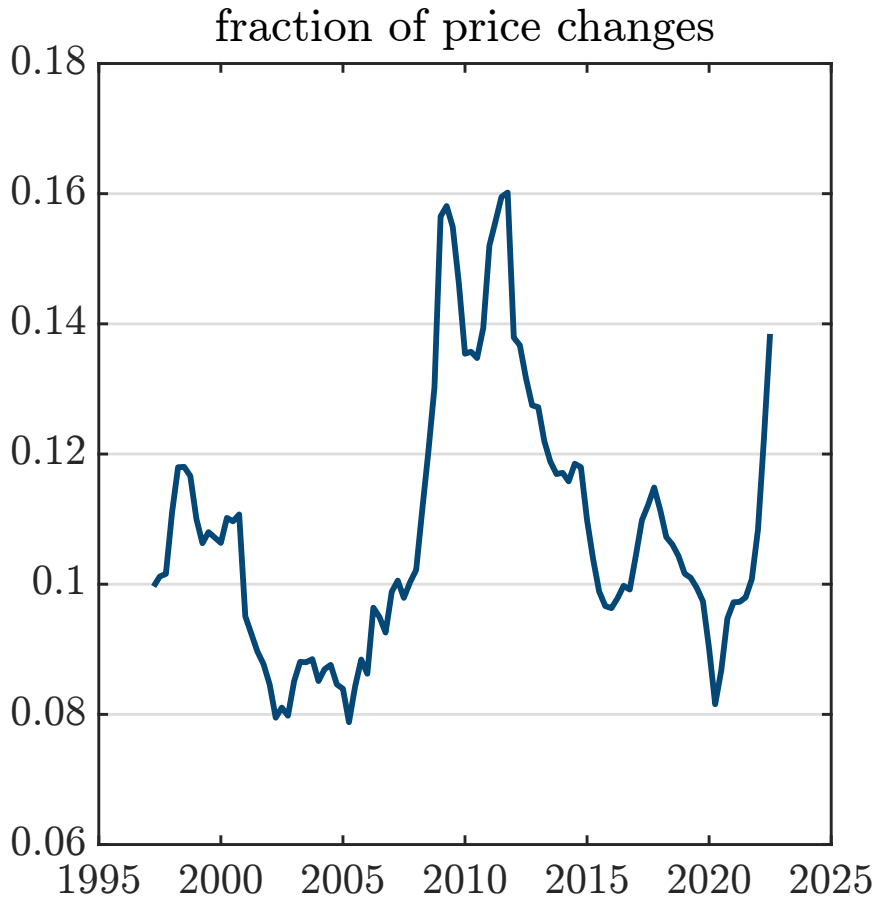




Source: Nakamura et al. (2018), Montag and Villar (2023)

Evidence from the UK





Source: Blanco, Boar, Jones, Midrigan (2024)

Question

How does the slope of the Phillips curve fluctuate over time?

- in models that reproduce comovement frequency and inflation

Existing Models

- Calvo model
  - widely used due to its tractability, constant fraction of price changes
- Menu cost model
  - less tractable: state of the economy includes distribution of prices
  - slope of Phillips curve depends on distribution of micro price changes
  - calibration consistent with micro data: fraction nearly constant
  - we propose a resolution, but curse of dimensionality persists
- Blanco, Boar, Jones, Midrigan (2024) model
  - multi-product firms choose *how many*, but not *which*, prices to change
  - exact aggregation: reduces to one-equation extension of Calvo

Our Findings

- Our model predicts highly non-linear Phillips curve
  - slope fluctuates from 0.02 in 1990s to 0.12 in 1970s – 1980s
- Mostly due to feedback loop between frequency and inflation
  - higher inflation increases frequency of price changes
  - higher frequency further increases inflation
  - *inflation accelerator*
- Absent feedback loop slope would increase to only 0.04 in 1970s – 1980s

Menu Cost Model

Overview

- Firms subject to
  - idiosyncratic quality shocks z_{it}
  - aggregate monetary shocks
- Labor-only technology
- Menu cost to change nominal price

Consumers

- Life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t \right)$$

- Budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t$$

- Monetary policy targets nominal spending $M_t = P_t c_t$

$$\log M_{t+1}/M_t = \mu_{t+1} \sim N(\mu, \sigma_m^2)$$

- Log-linear preferences imply $W_t = M_t$

Producers

- Technology

$$y_{it} = z_{it} l_{it}$$

- Differentiated varieties aggregated using

$$y_t = c_t = \left(\int_0^1 \left(\frac{y_{it}}{z_{it}} \right)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

- So demand for individual firm

$$y_{it} = z_{it} \left(\frac{z_{it} P_{it}}{P_t} \right)^{-\theta} y_t$$

- Aggregate price index

$$P_t \equiv \int_0^1 P_{it} \frac{y_{it}}{y_t} \mathrm{d}i = \left(\int_0^1 (z_{it} P_{it})^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

Shocks and Menu Costs

- Quality evolves according to

$$\log z_{it+1} = \log z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim N(0, \sigma^2)$$

- Menu cost $\bar{\xi}$ denominated in units of labor, so price change costs $W_t \bar{\xi}$
  - fixed cost: incur whenever $P_{it} \neq P_{it-1}$
  - so prices change infrequently, as in the data

Firm Objective

- Firm chooses P_{it} to

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\beta^t}{P_t c_t} \left((1+\tau) P_{it} y_{it} - W_t l_{it} - \bar{\xi} W_t \mathbb{I} \left(P_{it} \neq P_{it-1} \right) \right)$$

  - subsidy to eliminate markup distortion: $1+\tau = \frac{\theta}{\theta-1}$

- Define price gaps

$$x_{it} \equiv \frac{z_{it} P_{it}}{M_t} \quad \text{and} \quad x_t \equiv \left(\int x_{it}^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}} = \frac{P_t}{M_t}$$

- Firm objective

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(x_t^{\theta-1} \left((1+\tau) x_{it}^{1-\theta} - x_{it}^{-\theta} \right) - \bar{\xi} \mathbb{I}_{it} \right)$$

Equilibrium

- Firm state

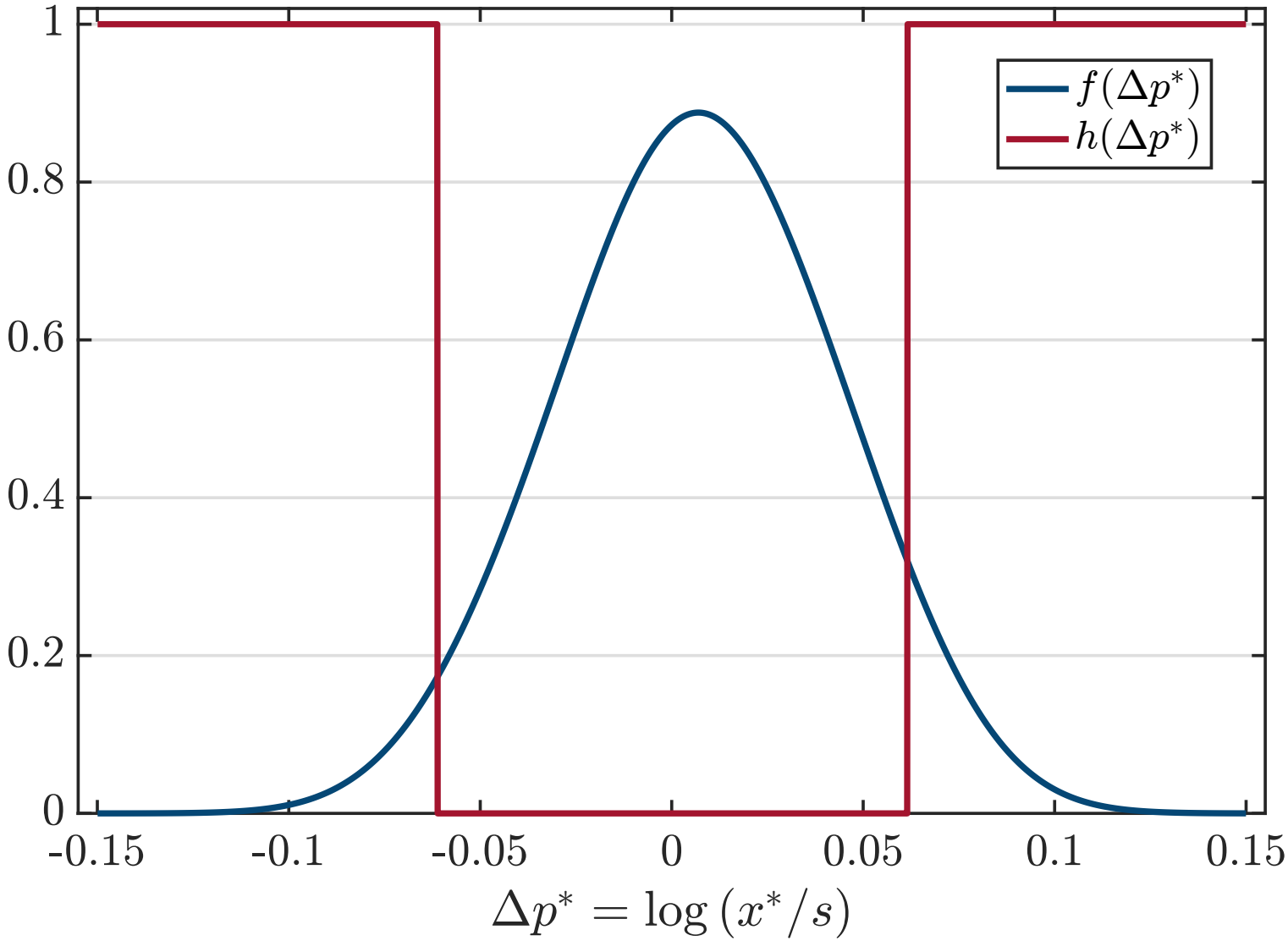
$$s_{it} = \frac{z_{it} P_{it-1}}{M_t} = x_{it-1} \exp(\varepsilon_{it} - \mu_t)$$

- Adjustment decision:
  - if don't adjust, $x_{it} = s_{it}$
  - if adjust, $x_{it} = x_t^*$

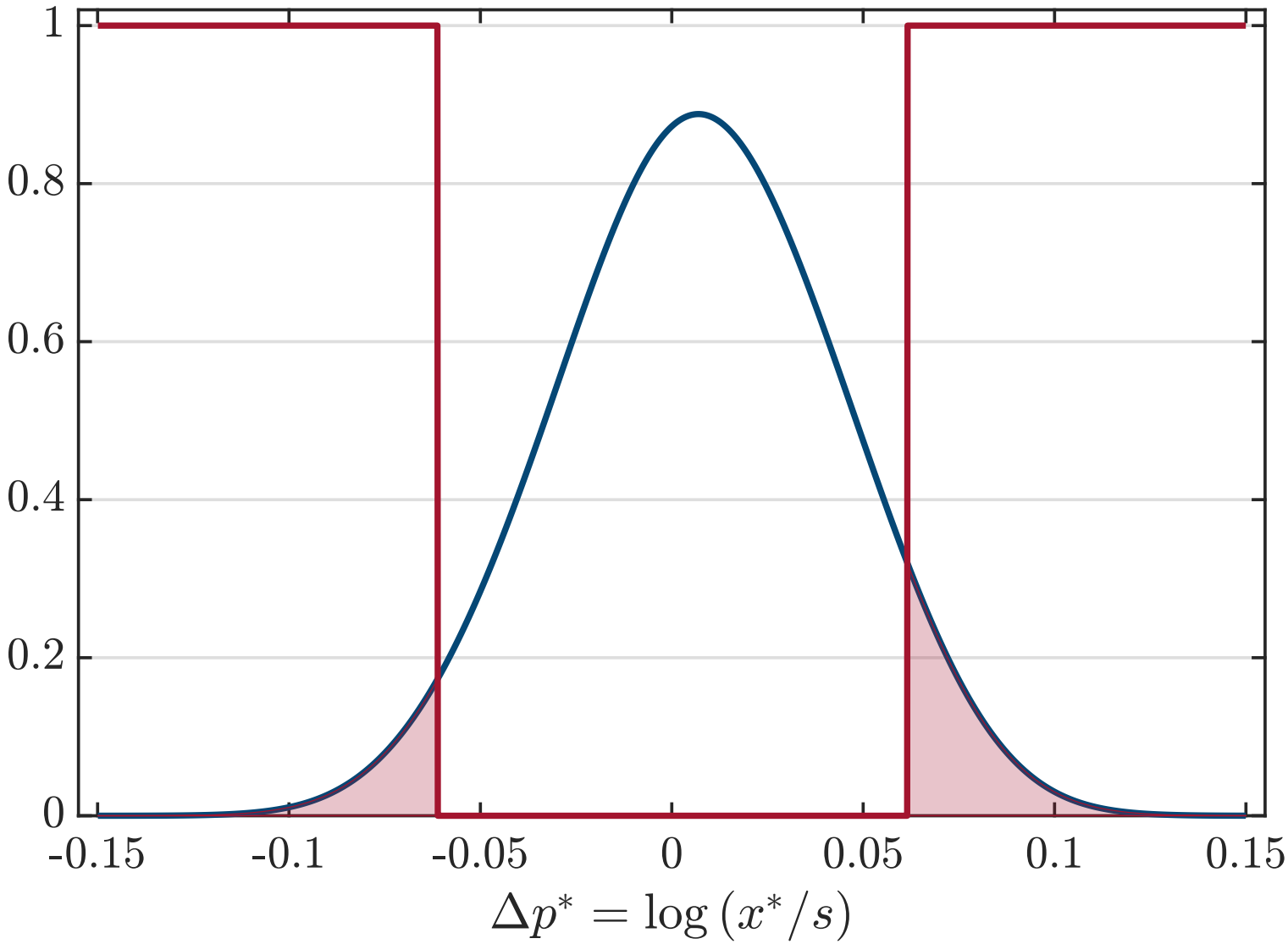
- Let $F_t(s)$ distribution of s_{it} and $h_t(s) \in \{0, 1\}$ adjustment hazard

$$x_t = \left(\int \left[h_t(s) \left(x_t^*\right)^{1-\theta} + \left(1 - h_t(s)\right) s^{1-\theta} \right] \mathrm{d}F_t(s) \right)^{\frac{1}{1-\theta}}$$

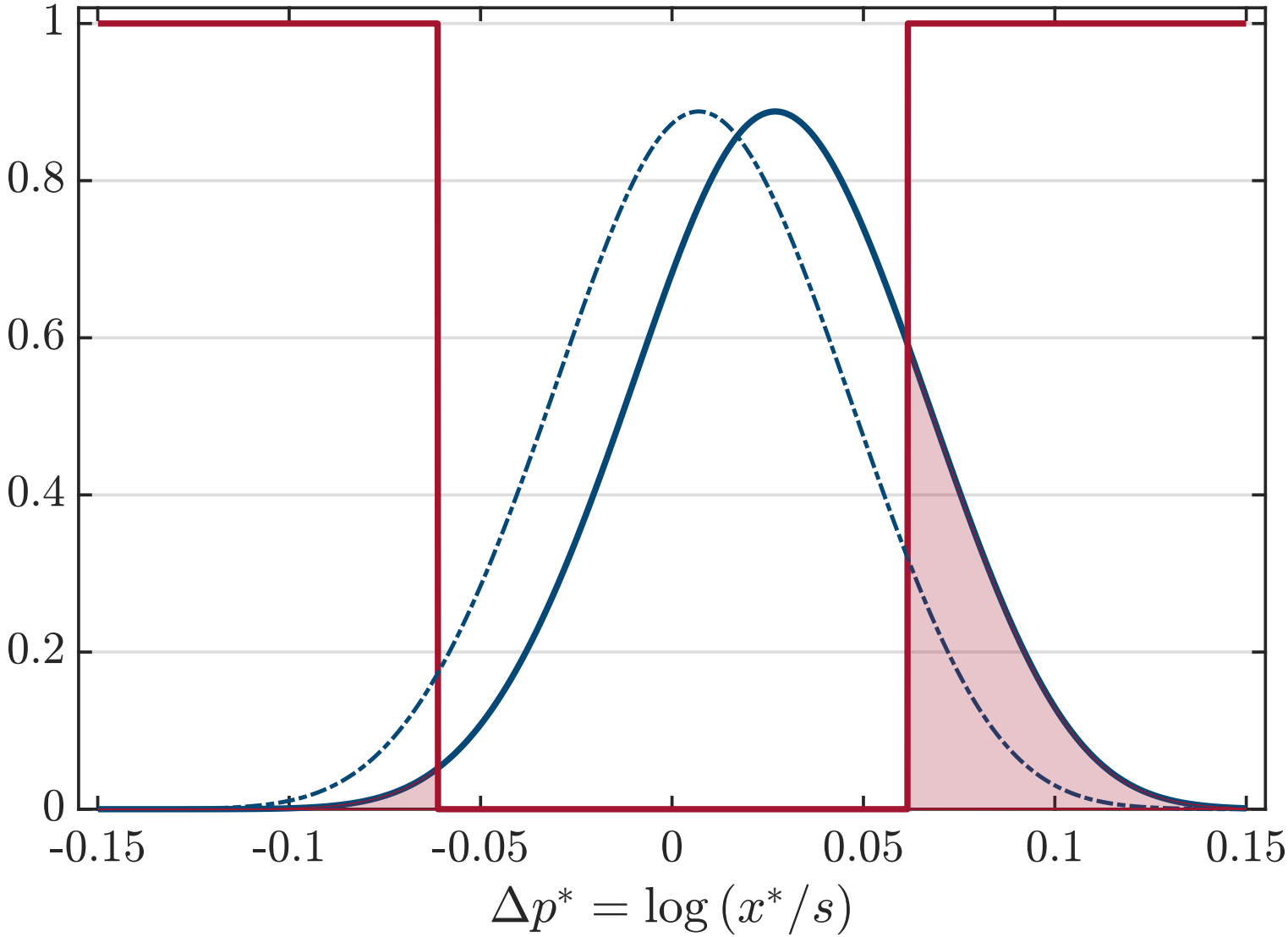
Distribution of Price Gaps and Hazard



Distribution of Price Changes



After a 2% Monetary Shock



Curse of Dimensionality

- Adjustment hazard non-linear so equilibrium x_t depends on $F_t(s)$
  - infinite-dimensional object so intractable problem

- Three approaches

  1. Krusell-Smith: project x_t on moments of $F_t(s)$

  2. impulse responses to one-time unanticipated shocks
     - difficult to account for state-dependence and non-linearities

  3. partial equilibrium with x exogenous
     - poor approximation with strategic complementarities

Importance of Non-Linearities

- Calibrate model to match micro-price statistics
- Use non-linear Krusell-Smith solution $x_t = \mathcal{X}(s_t)$
  - $s_t = \left(\int s^{1-\theta} \mathrm{d}F_t(s)\right)^{1-\theta} = x_{t-1}/\exp(\mu_t)$ moment of $F_t(s)$ distribution
- Back out money shocks μ_t that reproduce observed U.S. inflation
  - compare fraction of price changes predicted by model to data
  - evaluate importance of non-linearities in responses to monetary shocks

Parameterization

- Assigned parameters
  - period 1 month, $\beta = 0.96^{1/12}$, $\theta = 3$
- Calibrated parameters
  - average money growth rate μ and volatility σ_m
  - volatility of idiosyncratic shocks σ_z
  - menu cost $\bar{\xi}$
- Calibration targets (U.S., 1979 – 2014)
  - mean and volatility inflation
  - median size of a price change
  - mean fraction of price changes

Calibration

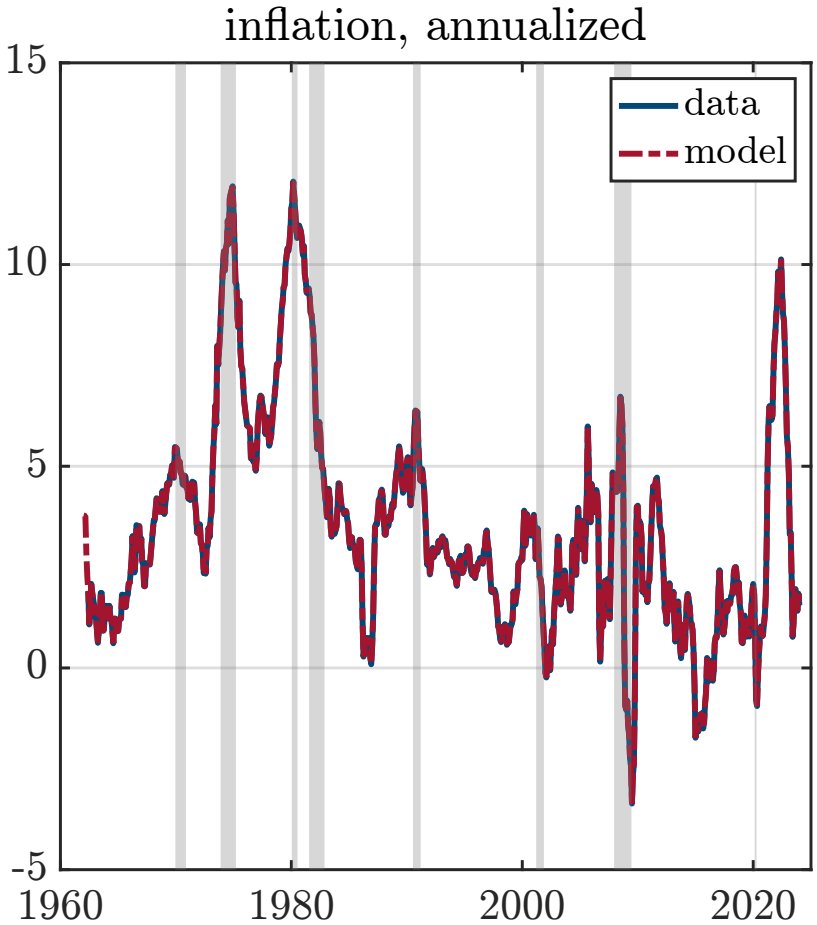
- Calibration targets

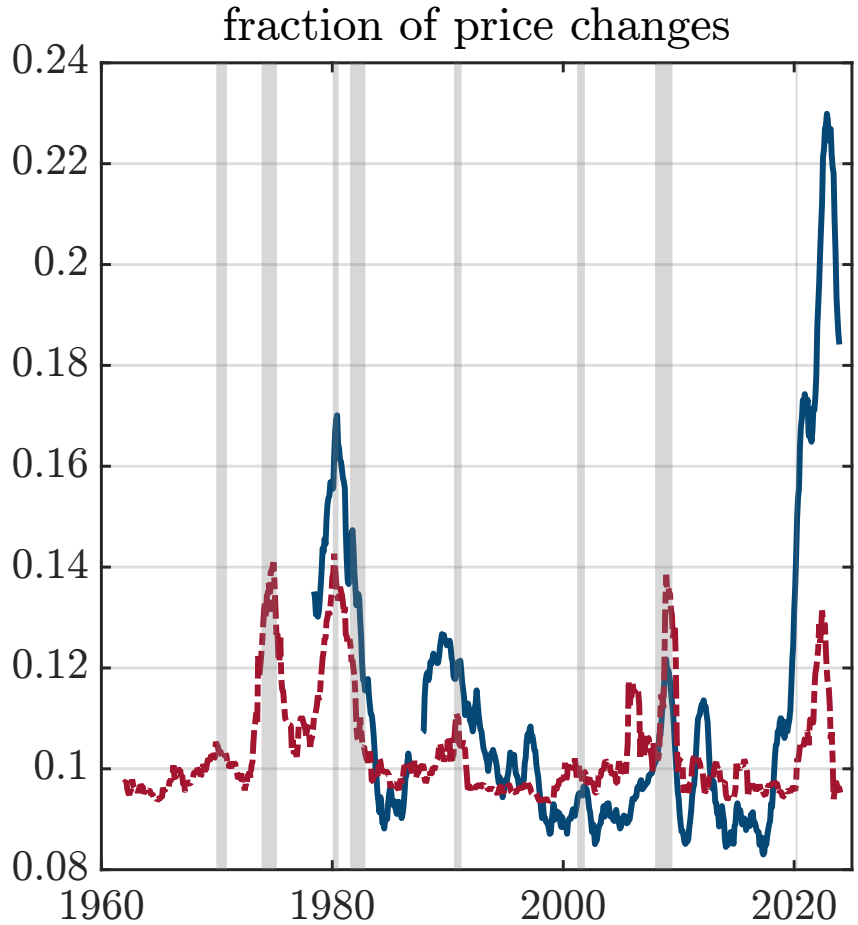
	Data	Model
mean π	0.034	0.034
std. dev. π	0.026	0.026
fraction Δp	0.105	0.105
median $\lvert\Delta p\rvert$	0.075	0.075

- Calibrated parameters

mean money growth	μ	0.034
volatility money growth	σ_m	0.008
volatility idios. shocks	σ_z	0.024
menu cost	$\bar{\xi}$	0.015

Fraction of Price Changes



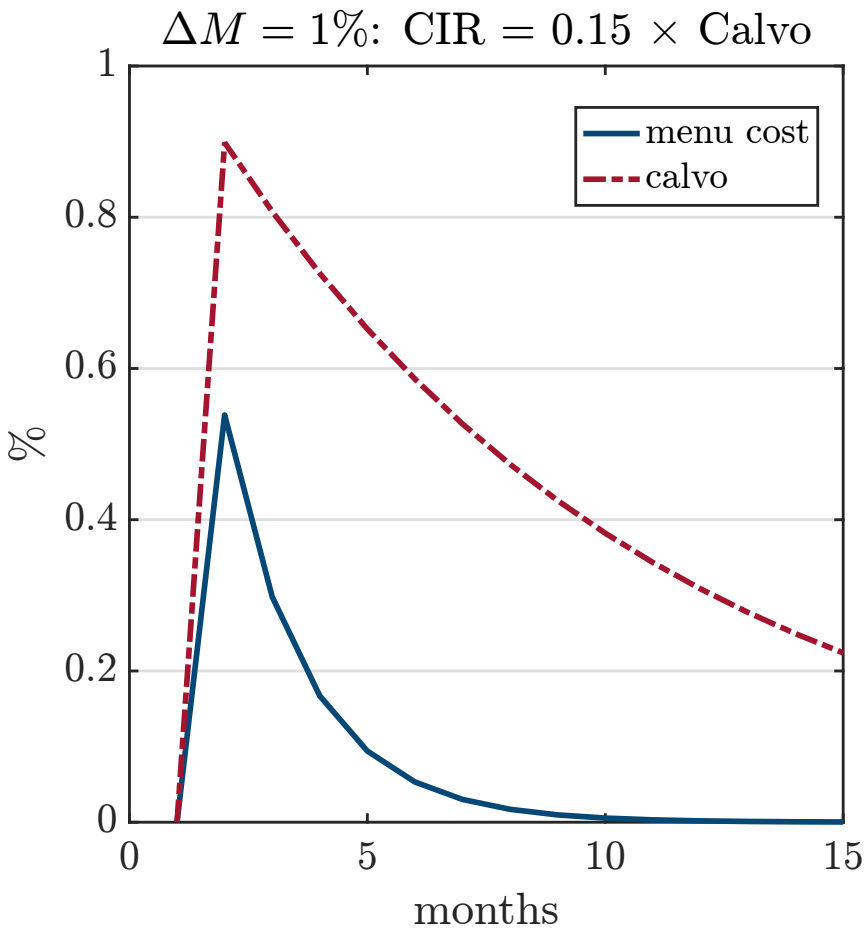


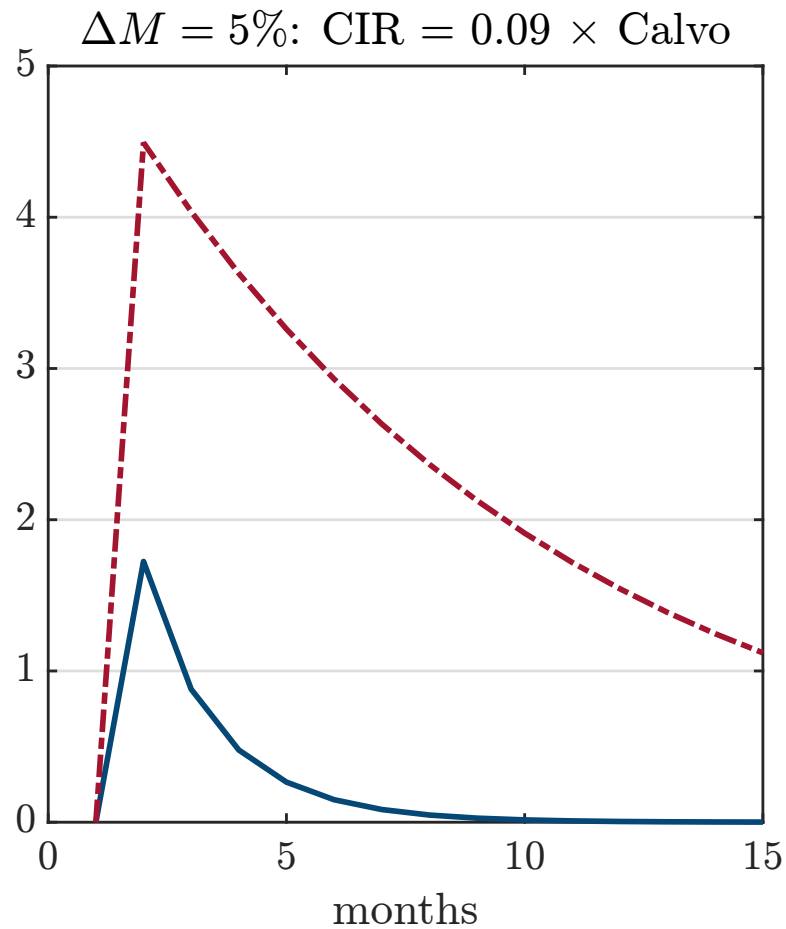
- Frequency increases with inflation. But only half as much as in the data
  - 0.14 vs. 0.17 in 1980, 0.13 vs. 0.23 in 2022

Real Effects of Monetary Policy Shocks

- Use Krusell-Smith solution to compute output response to one-time ΔM_t
  - $\Delta M_t = \Delta P_t + \Delta y_t$, so output response depends on how sticky prices are
- Show model features non-linearity
  - proportionally smaller output responses to larger shocks
  - due to increase in fraction of price changes
- However, even for small shocks, output response smaller than in Calvo
  - Golosov and Lucas result
  - aggregate price level flexible due to selection effect

Impulse Response to Monetary Shock

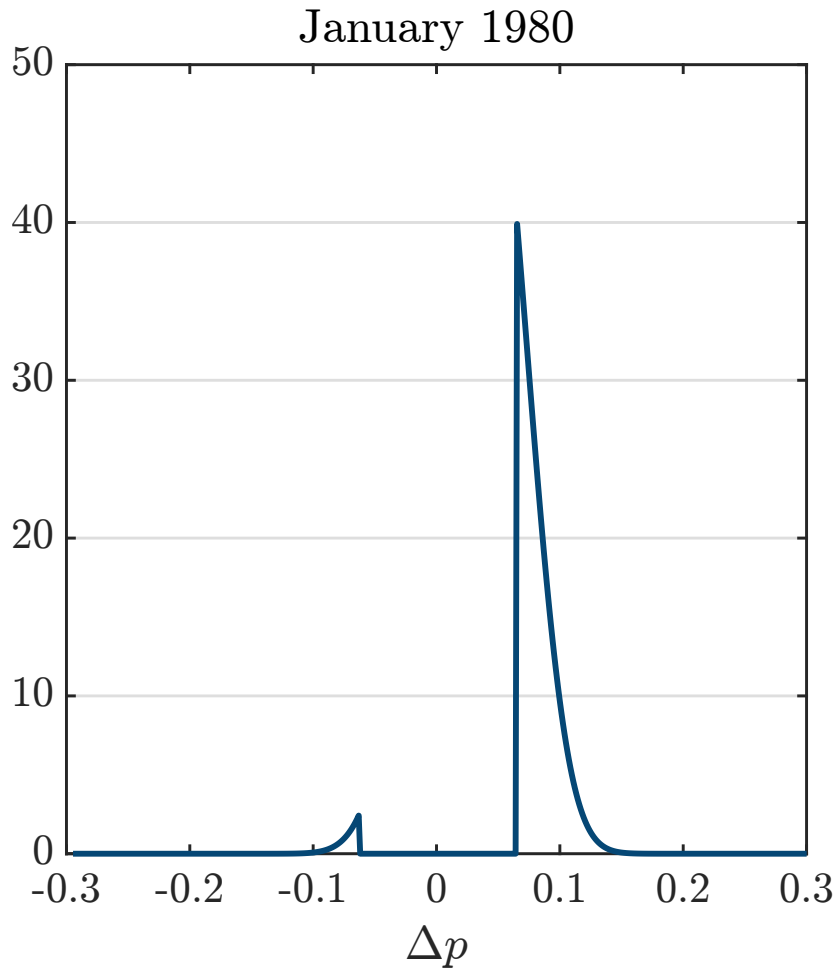


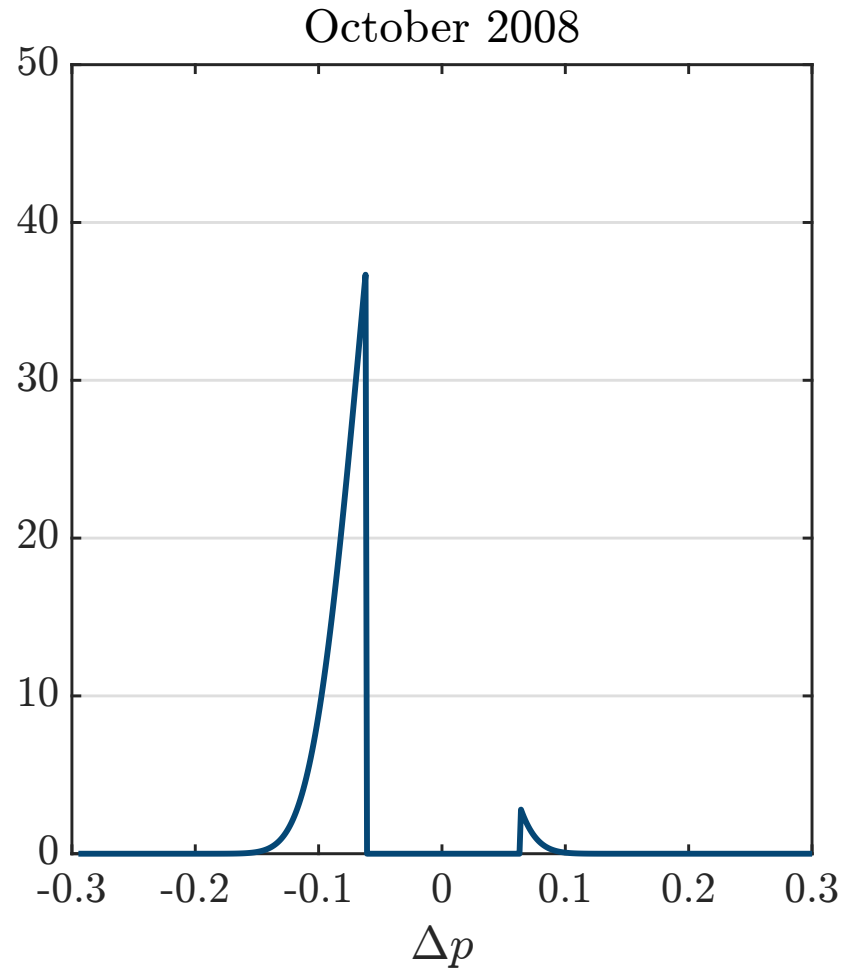


Selection Effect

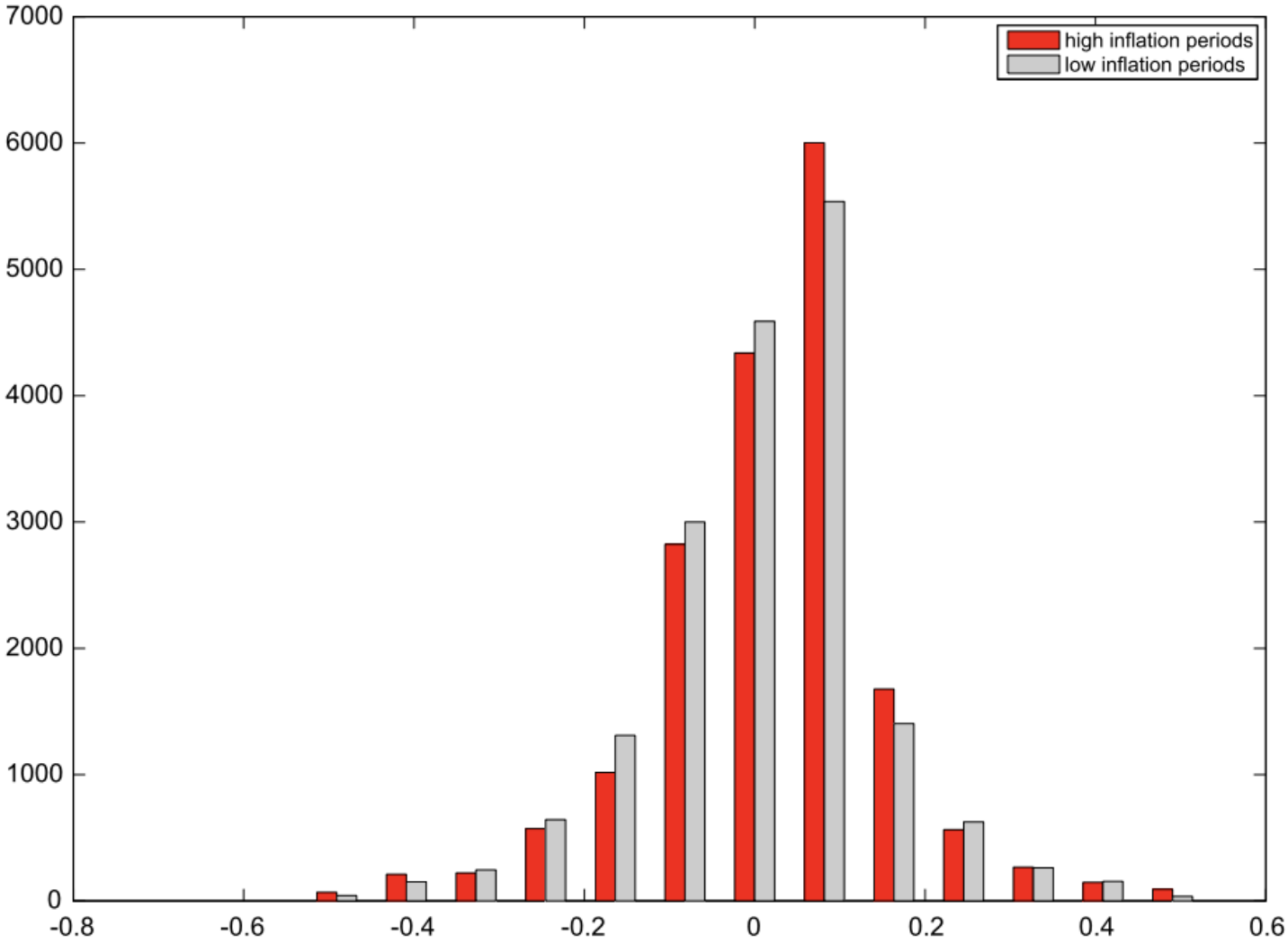
- Though responses non-linear, muted even for small shocks
- Due to *selection effect*
  - mix of adjusting firms changes in response to shock
  - since all change prices by large amounts, aggregate P flexible
- Selection effect strong because large mass of firms near adjustment cutoffs
- But model predicts too little dispersion in size of price changes

Distribution Δp in Golosov-Lucas



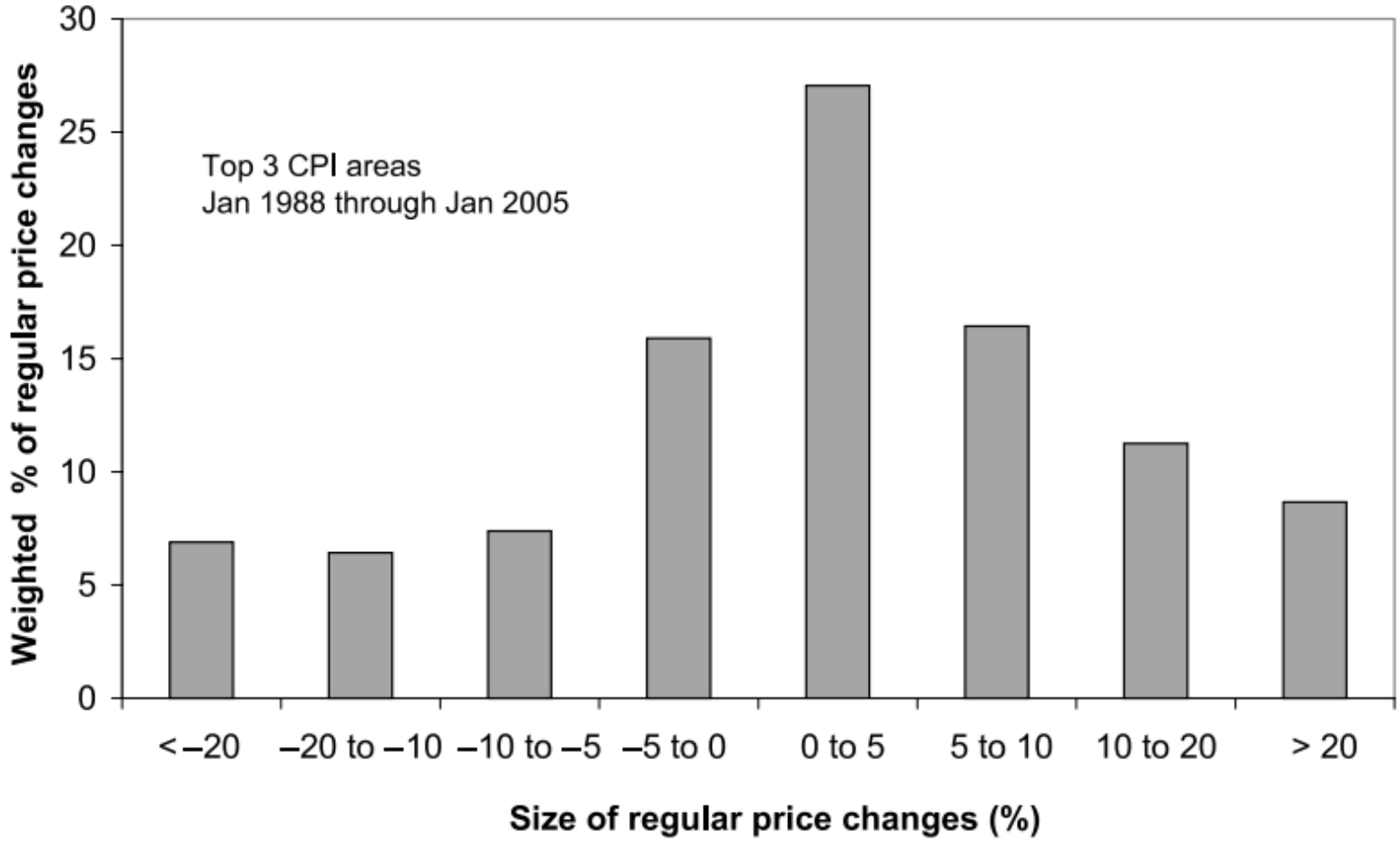


Greatly At Odds with Data



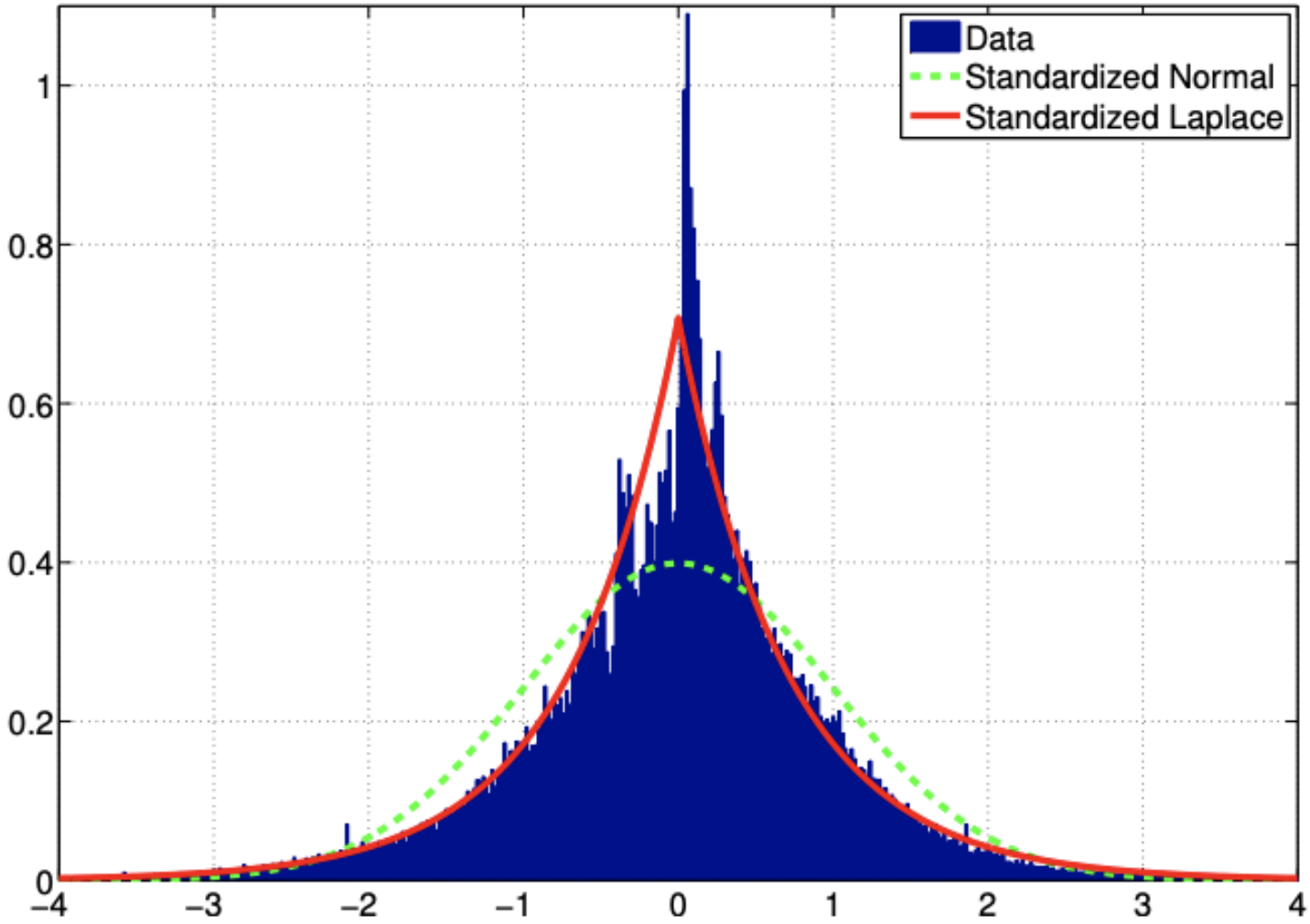
Source: Midrigan (2011) using data from US grocery store

Greatly At Odds with Data



Source: Klenow-Kryvtsov (2008) using US CPI data

Greatly At Odds with Data

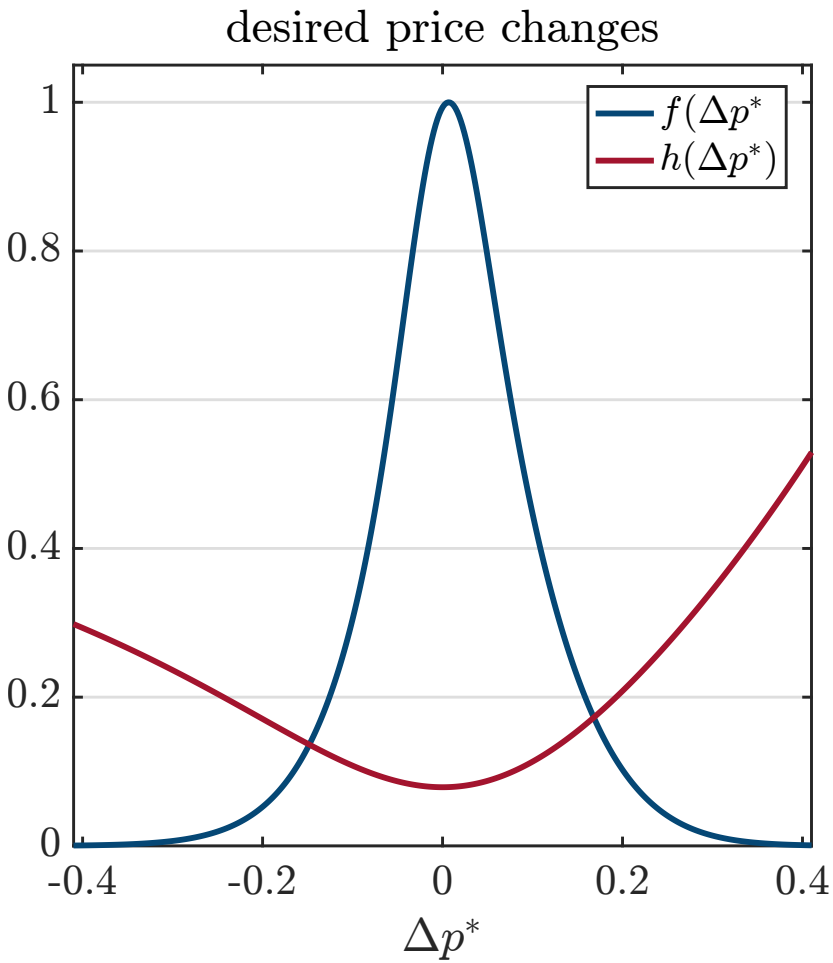


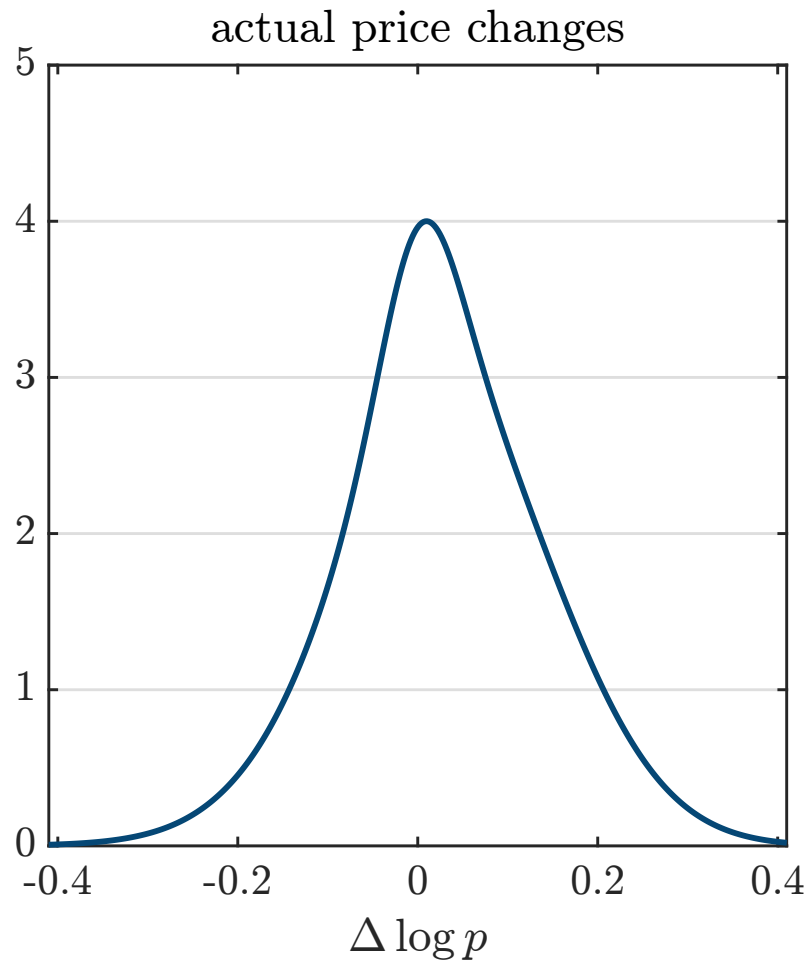
Source: Alvarez-Le Bihan-Lippi (2016) using French CPI data

Two Resolutions

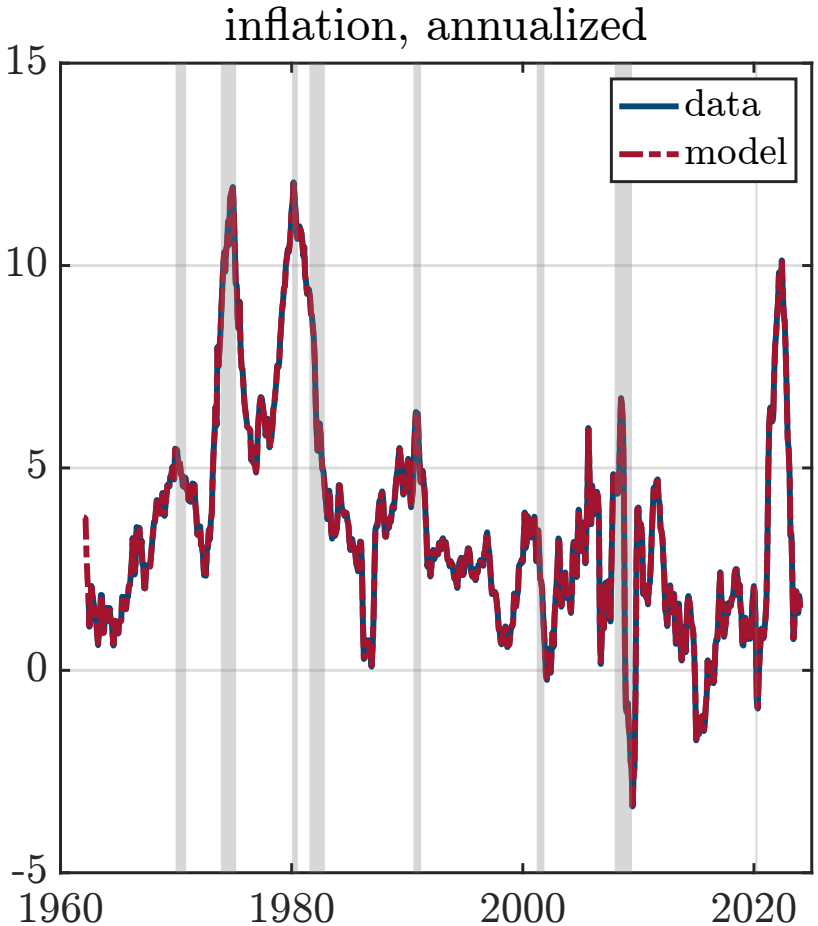
- Assume randomness in menu costs
  - illustrate by assuming 75% price changes free, otherwise $\xi_{it} \sim U\left[0, \bar{\xi}\right]$
- Multi-product firms with economies of scope in price setting
  - Midrigan 2011, Alvarez-Lippi 2014
  - pay a single menu cost to change *all* prices
  - if product-level shocks sufficiently uncorrelated, some price changes small
  - similar implications so skip today

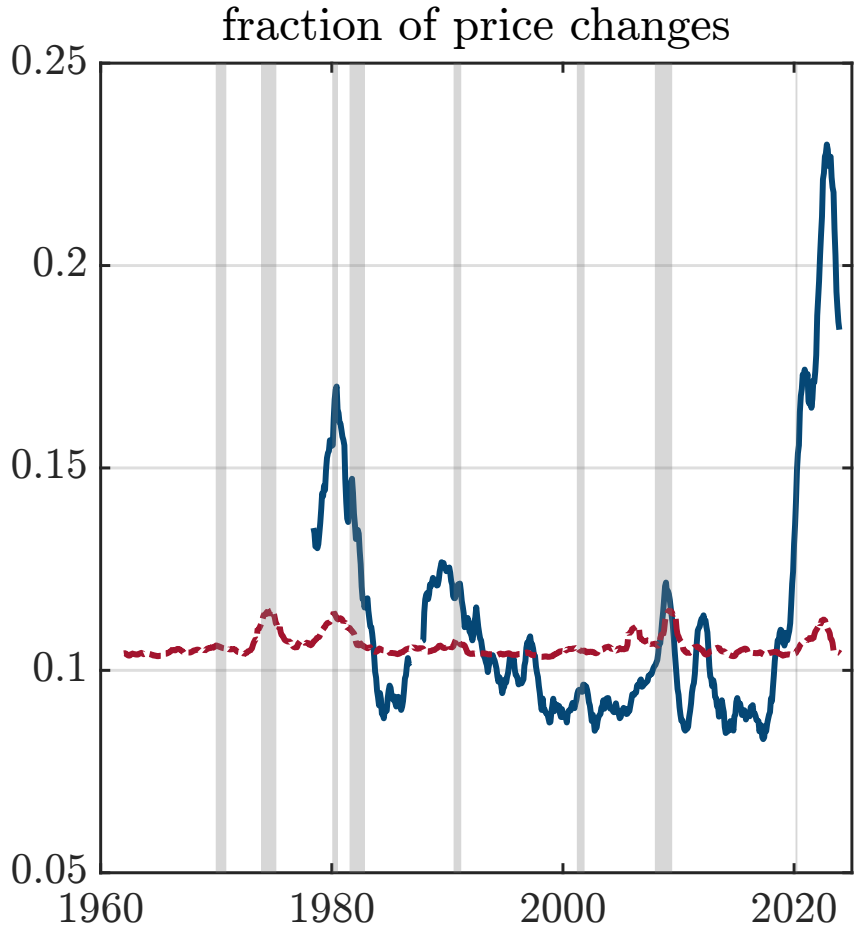
Distribution Δp with Random Menu Costs



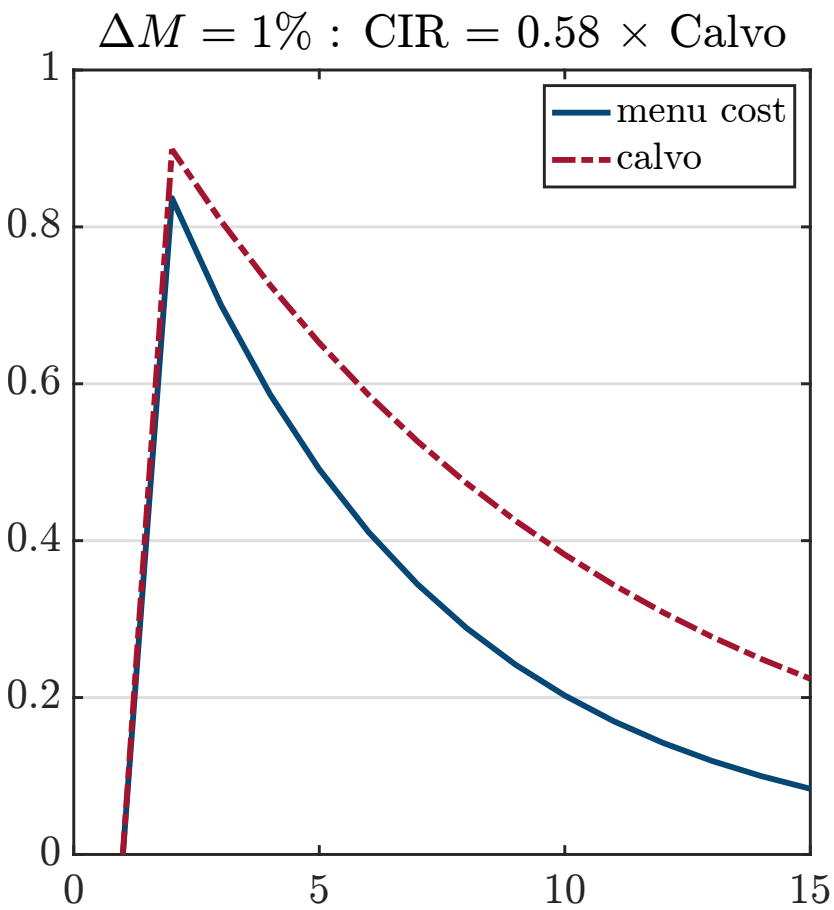


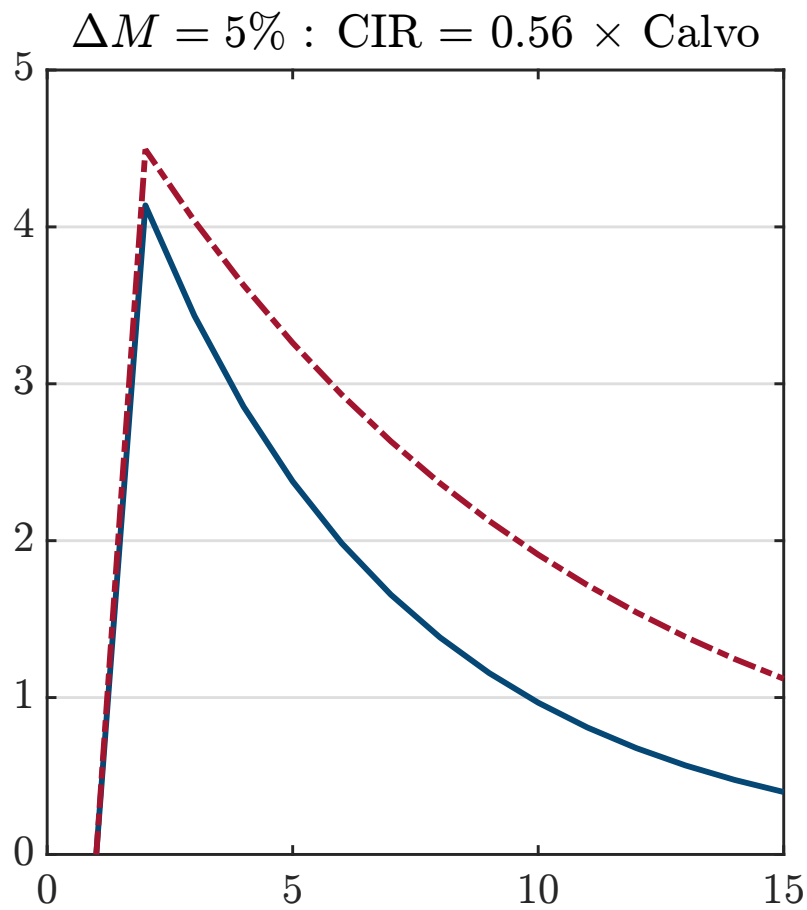
Fraction of Price Changes Nearly Constant





So Dynamics Nearly Linear





Summary

- Baseline menu cost model predicts some non-linearity
  - but fails to match dispersion in the size of price changes
  - and therefore predicts that the aggregate price level nearly flexible
- Random menu costs or economies of scope generate dispersion
  - but predict nearly constant fraction of price changes and no non-linearity
- Challenge: simultaneously account for both facts
  - BBJM (2024) address challenge using extension of multi-product model
  - but curse of dimensionality persists

The Inflation Accelerator

Model Overview

- Similar to menu cost model
- No idiosyncratic shocks since not critical here
- Introduce decreasing returns to scale to dampen slope of Phillips curve
- Firm sell continuum of products
  - choose *how many*, but *not which* price to change
  - subject to adjustment cost
  - admits exact aggregation, reduces to one-equation extension of Calvo

Technology

- Final good used for consumption, produced using CES aggregator

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

  - y_{ikt} output of good k produced by firm i, sold at price P_{ikt}

- Demand for individual product

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \qquad \text{where} \qquad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

- Individual goods produced with decreasing returns technology

$$y_{ikt} = (l_{ikt})^{\eta}$$

Price Adjustment Costs

- Real flow profits of firm i

$$\int_0^1 \left(\left(\frac{P_{ikt}}{P_t} \right)^{1-\theta} y_t - \tau \frac{W_t}{P_t} \left(\frac{P_{ikt}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}} \right) \mathrm{d}k$$

  - subsidy to eliminate markup distortion $\tau = 1 - 1/\theta$

- Firm chooses fraction of prices to change n_{it}

  - but not which prices to change

  - if adjust $P_{ikt} = P_{it}^*$, otherwise $P_{ikt} = P_{ikt-1}$

- Price adjustment cost, denominated in units of labor

$$\frac{\xi}{2} \left(n_{it} - \bar{n} \right)^2, \quad \text{if } n_{it} > \bar{n}$$

Aggregation

- Firm-level output y_{it} and labor l_{it}

$$y_{it} = \left(\int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \right)^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad l_{it} = \int_0^1 l_{ikt} \mathrm{d}k$$

- Firm production function

$$y_{it} = \left(\frac{X_{it}}{P_{it}} \right)^{\theta} l_{it}^{\eta}$$

- Firm price index P_{it} and losses from misallocation X_{it}

$$P_{it} = \left(\int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \right)^{\frac{1}{1-\theta}} \qquad \text{and} \qquad X_{it} = \left(\int_0^1 (P_{ikt})^{-\frac{\theta}{\eta}} \, \mathrm{d}k \right)^{-\frac{\eta}{\theta}}$$

- Absent price dispersion $X_{it}/P_{it} = 1$, otherwise $X_{it}/P_{it} < 1$

Firm Problem

- Choose reset price P_{it}^* and fraction of prices to change n_{it} to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\left(\frac{P_{it+s}}{P_{t+s}} \right)^{1-\theta} - \tau \left(\frac{X_{it+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} - \frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2 \right]$$

- Choices at t affect firm price index and misallocation at all future dates

$$\left(P_{it+s} \right)^{1-\theta} = n_{it+s} \left(P_{it+s}^* \right)^{1-\theta} + \left(1 - n_{it+s} \right) n_{it+s-1} \left(P_{it+s-1}^* \right)^{1-\theta} + \cdots$$

$$+ \prod_{j=1}^{s} \left(1 - n_{it+j} \right) n_{it} \left(P_{it}^* \right)^{1-\theta} + \prod_{j=1}^{s} \left(1 - n_{it+j} \right) \left(1 - n_{it} \right) \left(P_{it-1} \right)^{1-\theta}$$

▸ misallocation

- History encoded in two state variables: P_{it-1} and X_{it-1}
  - exact aggregation because adjustment hazard does not depend on P_{ikt-1}

Optimal Reset Price

- Present value of output and production costs in future dates

$$b_{1it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t} \right)^{\theta - 1}$$

$$b_{2it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t} \right)^{\frac{\theta}{\eta}} (y_{t+s})^{\frac{1}{\eta}}$$

- Optimal reset price

$$\frac{P_{it}^*}{P_t} = \left(\frac{1}{\eta} \frac{b_{2it}}{b_{1it}} \right)^{\frac{1}{1+\theta\left(\frac{1}{\eta} - 1\right)}}$$

- Present value of future marginal costs, weighted appropriately
  - similar to Calvo, except n_{it} time-varying

Optimal Fraction of Price Changes

- Equate marginal cost to marginal benefit

$$\xi\left(n_{it}-\bar{n}\right)=b_{1it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{1-\theta}-\left(\frac{P_{it-1}}{P_{t}}\right)^{1-\theta}\right)-\tau b_{2it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{it-1}}{P_{t}}\right)^{-\frac{\theta}{\eta}}\right)$$

- Marginal benefit: higher n_{it}
  - changes firm price index
  - and reduces misallocation
  - weighted by the same terms b_{1it} and b_{2it} that determine P_{it}^{*}

Symmetric Equilibrium

- Since firms are identical, in equilibrium $P_{it}^* = P_t^*$, $n_{it} = n_t$, ...

- Let $p_t = P_t/M_t$, $p_t^* = P_t^*/M_t$, $x_t = X_t/P_t$, $\pi_t = P_t/P_{t-1}$

- Optimal choices

  - reset price

$$\frac{p_t^*}{p_t} = \left(\frac{1}{\eta}\frac{b_{2t}}{b_{1t}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$$

  - fraction of price changes

$$\xi\left(n_t - \bar{n}\right) = b_{1t}\left(\left(\frac{p_t^*}{p_t}\right)^{1-\theta} - \left(\frac{1}{\pi_t}\right)^{1-\theta}\right) - \tau b_{2t}\left(\left(\frac{p_t^*}{p_t}\right)^{-\frac{\theta}{\eta}} - \left(\frac{x_{t-1}}{\pi_t}\right)^{-\frac{\theta}{\eta}}\right)$$

Symmetric Equilibrium

- Inflation pinned down by definition of price index

$$1 = n_t \left(\frac{p_t^*}{p_t} \right)^{1-\theta} + (1 - n_t)\, \pi_t^{\theta-1}$$

- Losses from misallocation

$$x_t^{-\frac{\theta}{\eta}} = n_t \left(\frac{p_t^*}{p_t} \right)^{-\frac{\theta}{\eta}} + (1 - n_t)\, x_{t-1}^{-\frac{\theta}{\eta}} \pi_t^{\frac{\theta}{\eta}}$$

- Model reduces to one-equation extension of Calvo
  - as $\xi \to \infty$, $n_t = \bar{n}$ so our model nests Calvo

- Solved with global methods, but 3rd order approx. very accurate

Calibration

- Assigned parameters
  - period 1 quarter so $\beta = 0.99$
  - demand elasticity $\theta = 6$ and span of control $\eta = 2/3$
- Calibrated parameters
  - mean and standard deviation of nominal spending growth μ and σ
  - fraction of free price changes $\bar{n}$
  - price adjustment cost ξ
- Calibration targets
  - mean and standard deviation of inflation
  - mean fraction of price changes
  - slope of fraction of price changes on absolute value of inflation

Calibrated Parameters

Targeted Moments

	Data	Our model	Calvo
mean inflation	3.291	3.291	3.291
s.d. inflation	2.313	2.313	2.313
mean fraction	0.294	0.294	0.294
slope of n_t on $\lvert\pi_t\rvert$	0.019	0.019	–

Calibrated Parameters

		Our model	Calvo
μ	mean spending growth rate	0.033	0.034
σ	s.d. monetary shocks	0.018	0.020
$\bar{n}$	fraction free price changes	0.232	0.294
ξ	adjustment cost	1.233	–

Inflation Response to Shocks

- Perturb around equilibrium conditions in period t
  - hats denote log-deviations

- Expression for aggregate price index

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\,\pi^{\theta-1}}\,\frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\,\hat{n}_t + \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}\,(\hat{p}_t^* - \hat{p}_t)$$

- Elasticity $\mathcal{N}_t$ to reset price changes: identical to Calvo
  - increases with n_t, decreases with π_t (lower weight on new prices)

- Elasticity $\mathcal{M}_t$ to frequency: zero if $\pi_t = 1$, increases with inflation
  - inflation = fraction of price changes $\times$ average price change

Fraction Response to Shocks

- Log-linearized expression for optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}_t \hat{\pi}_t + \mathcal{B}_t \left(\hat{p}_t^* - \hat{p}_t \right) - \mathcal{C}_t \hat{x}_{t-1} + \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

- Elasticities increase with inflation

$$\mathcal{A}_t = \frac{\theta - 1}{\xi n_t} b_{1t} \left(\left(\frac{p_t^*}{p_t} \right)^{1+\theta\left(\frac{1}{\eta}-1\right)} \pi_t^{\frac{\theta}{\eta}} \left(x_{t-1} \right)^{-\frac{\theta}{\eta}} - \pi_t^{\theta-1} \right)$$

$$\mathcal{B}_t = \left(1 - \tau\eta \right) \left(\theta - 1 \right) \frac{b_{1t}}{\xi n_t} \left(\left(\frac{p_t^*}{p_t} \right)^{1+\theta\left(\frac{1}{\eta}-1\right)} \pi_t^{\frac{\theta}{\eta}} \left(x_{t-1} \right)^{-\frac{\theta}{\eta}} - \left(\frac{p_t^*}{p_t} \right)^{1-\theta} \right)$$

Slope of the Phillips Curve

- Slope of the Phillips curve

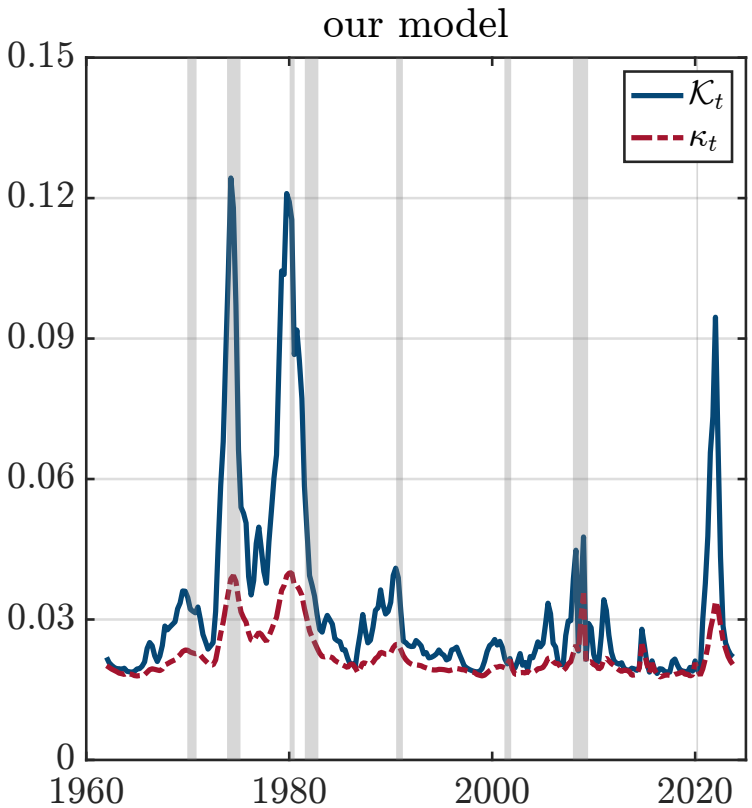
$$\mathcal{K}_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}$$

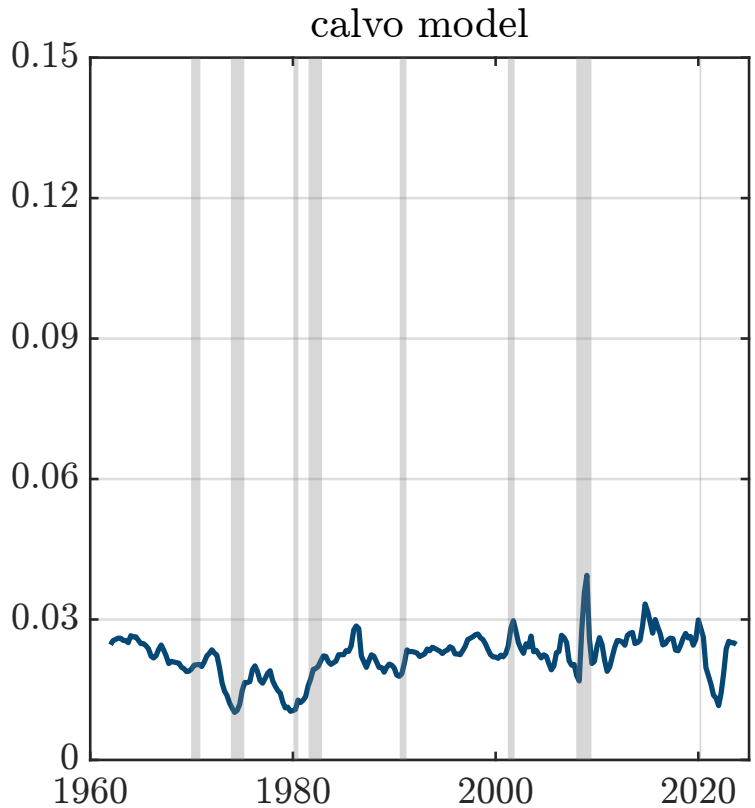
- Absent endogenous frequency response

$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\mathcal{N}_t$$

- Difference $\mathcal{K}_t - \kappa_t$ captures *inflation accelerator*
  - more inflation leads to more price changes which leads to more inflation ...

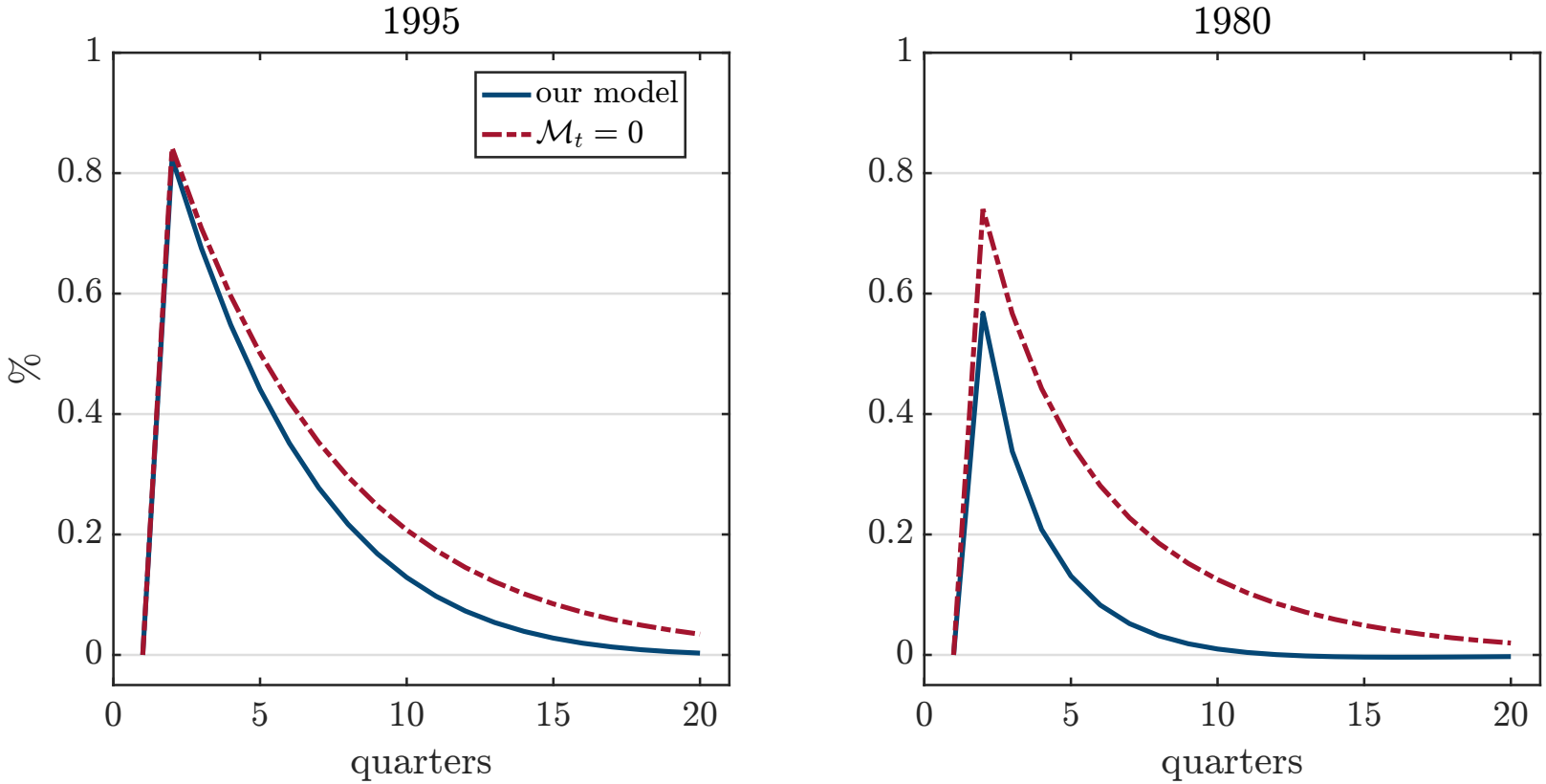
Time-Varying Slope of the Phillips Curve





Ranges from 0.02 to 0.12, mostly due to inflation accelerator, opposite of Calvo

Implications

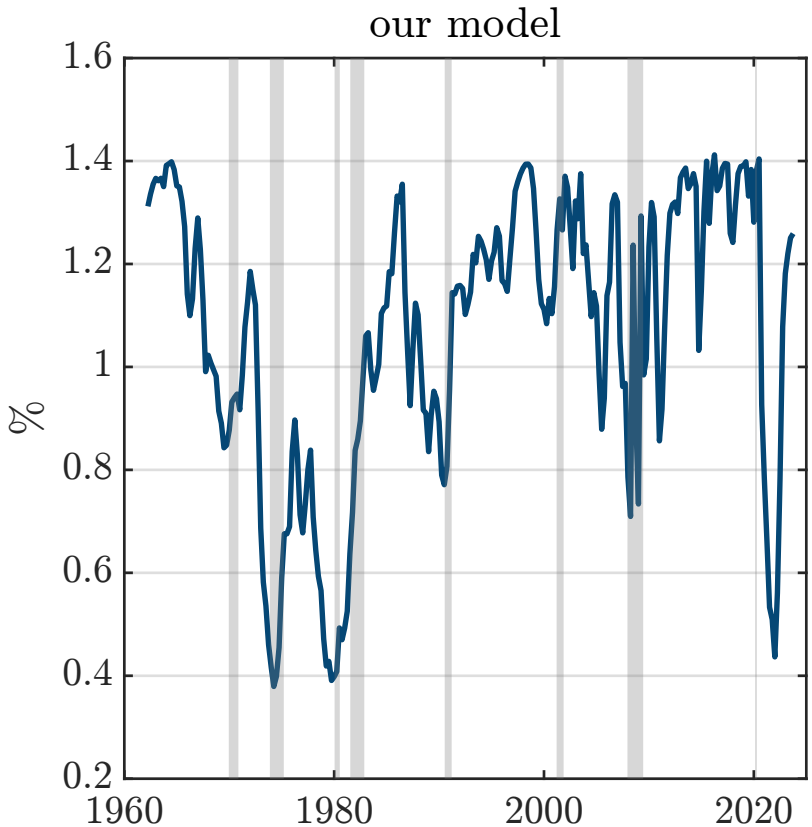


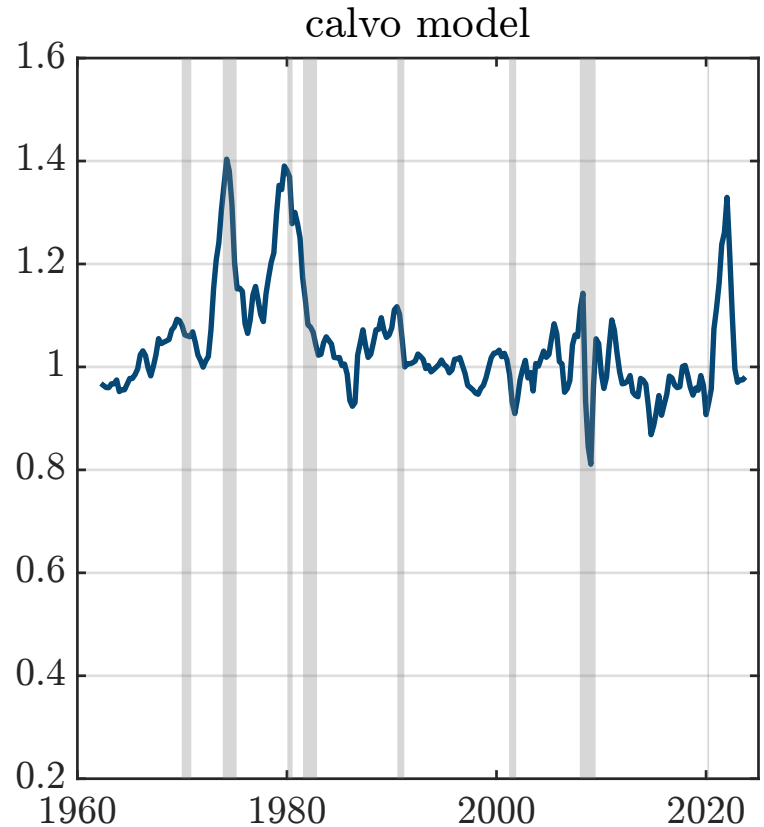
Less monetary non-neutrality in periods of high inflation

Sacrifice Ratio

- Time-varying slope: reducing inflation less costly when inflation is high
- Calculate average drop in output needed to reduce π by 1% over a year
  - use non-linear solution of the model
  - report average decline in output over 4 quarters of that year

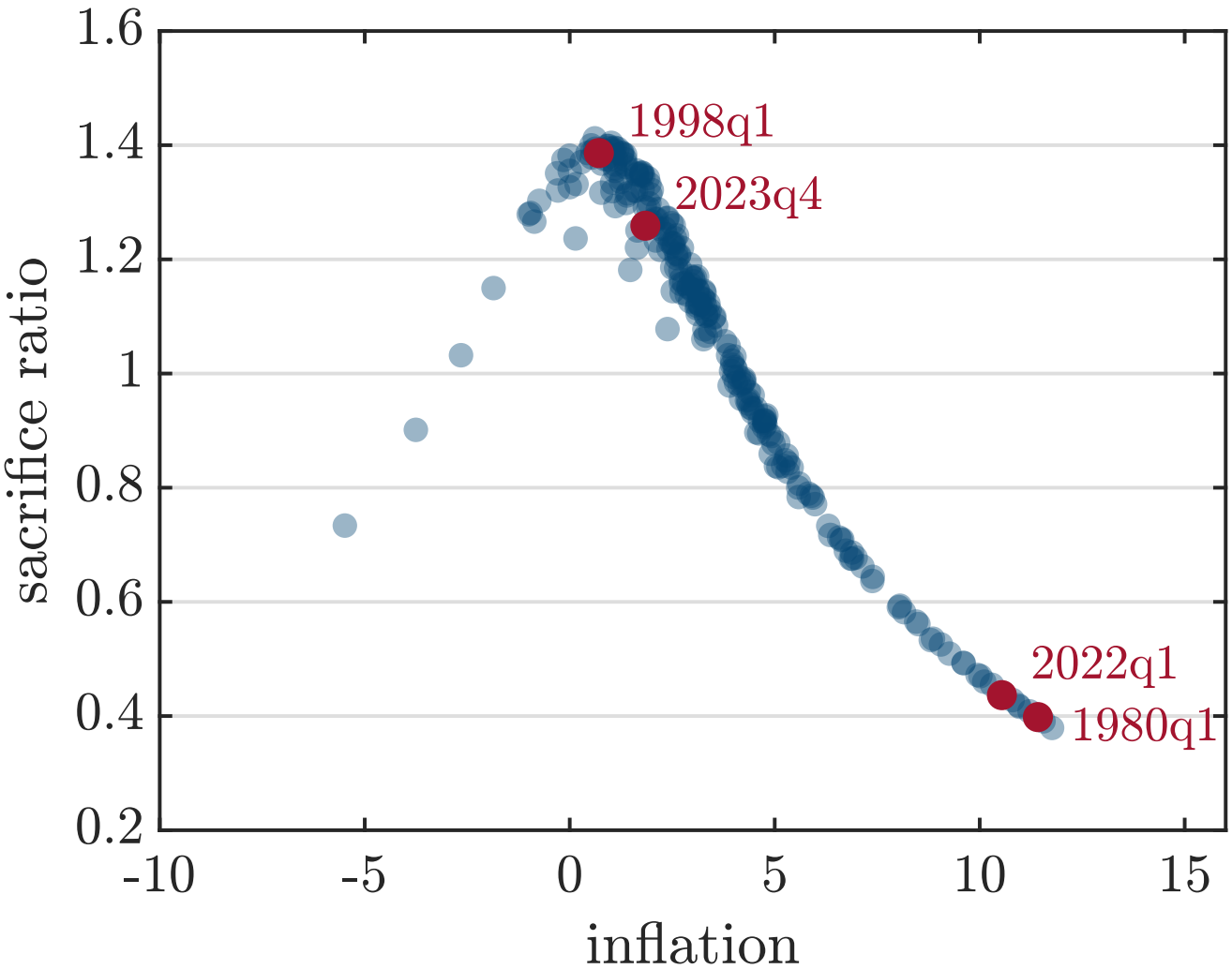
Sacrifice Ratio





Ranges from 0.4% (high inflation) to 1.4% (low inflation), opposite of Calvo

Inflation and the Sacrifice Ratio



Conclusions

- Data: fraction of price changes rises with inflation

- Menu cost models consistent with micro data cannot reproduce this
  - provide resolution using multi-product model
  - but subject to curse of dimensionality

- Developed tractable model consistent with this evidence
  - firms choose how many, but not which prices to change

- Predicts slope of Phillips curve increases considerably with inflation
  - primarily due to feedback loop frequency/inflation– *inflation accelerator*